21

SURFACE AREAS AND VOLUMES OF SOLID FIGURES

In the previous lesson, you have studied about perimeters and areas of plane figures like rectangles, squares, triangles, trapeziums, circles, sectors of circles, etc. These are called plane figures because each of them lies wholly in a plane. However, most of the objects that we come across in daily life do not wholly lie in a plane. Some of these objects are bricks, balls, ice cream cones, drums, and so on. These are called solid objects or three dimensional objects. The figures representing these solids are called **three dimensional** or **solid figures**. Some common solid figures are cuboids, cubes, cylinders, cones and spheres. In this lesson, we shall study about the surface areas and volumes of all these solids.

OBJECTIVES

After studying this lesson, you will be able to

- *explain the meanings of surface area and volume of a solid figure,*
- *identify situations where there is a need of finding surface area and where there is a need of finding volume of a solid figure;*
- *find the surface areas of cuboids, cubes, cylinders, cones spheres and hemispheres, using their respective formulae;*
- *find the volumes of cuboids, cubes, cylinders, cones, spheres and hemispheres using their respective formulae;*
- *solve some problems related to daily life situations involving surface areas and volumes of above solid figures.*

EXPECTED BACKGROUND KNOWLEDGE

- Perimeters and Areas of Plane rectilinear figures.
- Circumference and area of a circle.

- Four fundamental operations on numbers
- Solving equations in one or two variables.

21.1 MEANINGS OF SURFACE AREA AND VOLUME

Look at the following objects given in Fig. 21.1.

Fig. 21.1

Geometrically, these objects are represented by three dimensional or solid figures as follows:

Objects	**Solid Figure**
Bricks, Almirah	Cuboid
Die, Tea packet	Cube
Drum, powder tin	Cylinder
Jocker's cap, Icecream cone,	Cone
Football, ball	Sphere
Bowl.	Hemisphere

You may recall that a rectangle is a figure made up of only its sides. You may also recall that the sum of the lengths of all the sides of the rectangle is called its permeter and the measure of the region enclosed by it is called its **area**. Similarly, the sum of the lengths of the three sides of a triangle is called its **permeters**, while the measure of the region enclosed by the triangle is called its area. In other words, the measure of the plane figure, i.e., the boundary triangle or rectangle is called its perimeter, while the measure of the plane region enclosed by the figure is called its **area**.

Following the same analogy, a solid figure is made up of only its boundary (or outer surface). For example, cuboid is a solid figure made up of only its six rectangular regions (called its faces). Similarly, a sphere is made up only of its outer surface or boundary. Like plane figures, solid figures can also be measured in two ways as follows:

(1) Measuring the surface (or boundary) constituting the solid. It is called the **surface area** of the solid figure.

(2) Measuring the space region enclosed by the solid figure. It is called the **volume** of the solid figure.

Thus, it can be said that the surface area is the measure of the solid figure itself, while volume is the measure of the space region enclosed by the solid figure. Just as area is measured in square units, volume is measured in **cubic units**. If the unit is chosen as a **unit cube** of side 1 cm, then the unit for volume is cm^3, if the unit is chosen as a **unit cube** of side 1m, then the unit for volume is m^3 and so on.

In daily life, there are many situations, where we have to find the surface area and there are many situations where we have to find the volume. For example, if we are interested in white washing the walls and ceiling of a room, we shall have to find the surface areas of the walls and ceiling. On the other hand, if we are interested in storing some milk or water in a container or some food grains in a godown, we shall have to find the volume.

21.2 CUBOIDS AND CUBES

As already stated, a brick, chalk box, geometry box, match box, a book, etc are all examples of a cuboid. Fig. 21.2 represents a cuboid. It can be easily seen from the figure that a cuboid has six rectangular regions as its faces. These are ABCD, ABFE, BCGF, EFGH, ADHE and CDHG.. Out of these, opposite faces ABFE and CDHG; ABCD and EFGH and ADHE and BCGH are respectively congruent and parallel to each other. The two adjacent faces meet in a line segment called an **edge** of the cuboid. For example, faces ABCD and ABFE meet in the **edge** AB. There are in all 12 edges of a cuboid. Points A,B,C,D,E,F,G and H are called the **corners** or **vertices** of the cuboid. So, there are 8 **corners** or **vertices** of a cuboid.

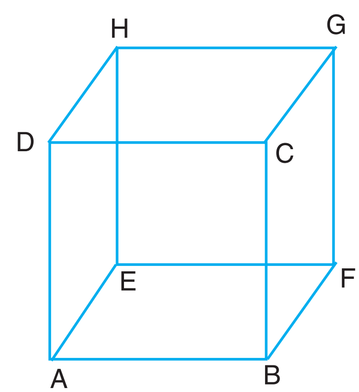


Fig. 21.2

It can also be seen that at each vertex, three edges meet. One of these three edges is taken as the length, the second as the breadth and third is taken as the height (or thickness or depth) of the cuboid. These are usually denoted by l, b and h respectively. Thus, we may say that A B (= EF = CD = GH) is the **length**, AE (=BF = CG = DH) is the **breadth** and AD (= EH = BC = FG) is the **height** of the cuboid.

Note that three faces ABFE, AEHD and EFGH meet at the vertex E and their opposite faces DCGH, BFGC and ABCD meet at the point C. Therefore, E and C are called the **opposite corners** or **vertices** of the cuboid. The line segment joing E and C. i.e., EC is called a **diagonal** of the cuboid. Similarly, the diagonals of the cuboid are AG, BH and FD. In all there are **four diagonals** of cuboid.

Surface Area

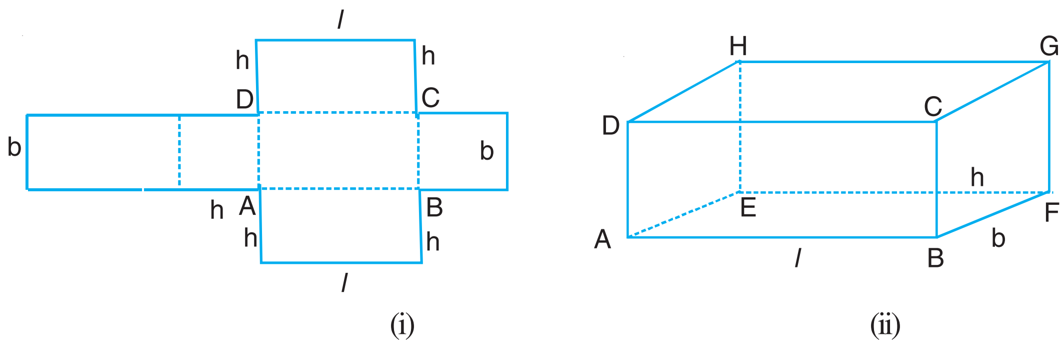


Fig. 21.3

Look at Fig. 21.3 (i). If it is folded along the dotted lines, it will take the shape as shown in Fig. 21.3 (ii), which is a cuboid. Clearly, the length, breadth and height of the cuboid obtained in Fig. 21.3 (ii) are l, b and h respectively. What can you say about its surface area. Obviously, surface area of the cuboid is equal to the sum of the areas of all the six rectangles shown in Fig. 21.3 (i).

Thus, surface area of the cuboid

$= l\times b + b\times h + h\times l + l\times b + b\times h + h\times l$

$= 2(lb + bh + hl)$

In Fig. 21.3 (ii), let us join BE and EC (See Fig. 21.4)

We have :

$BE^2 = AB^2 + AE^2$ (As $\angle EAB = 90^o$)

or $BE^2 = l^2 + b^2$ -(1)

Also, $EC^2 = BC^2 + BE^2$ (As $\angle CBE = 90^o$)

or $EC^2 = h^2 + l^2 + b^2$ [From (i)]

So, $EC = \sqrt{l^2 + b^2 + h^2}$.

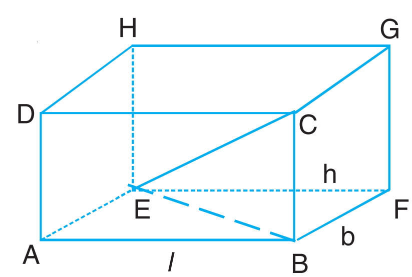


Fig. 21.4

Hence, **diagonal of a cuboid** = $\sqrt{l^2 + b^2 + h^2}$.

We know that cube is a special type of cuboid in which length = breadth = height, i.e., l = b = h.

Hence,

surface area of a cube of side or edge a

$$= 2\,(a\times a + a\times a + a\times a)$$

$$= 6a^2$$

and its **diagonal** = $\sqrt{a^2 + a^2 + a^2}. = a\sqrt{3}.$

Note: Fig. 21.3 (i) is usually referred to as a **net** of the cuboid given in Fig. 21.3 (ii).

Volume:

Take some unit cubes of side 1 cm each and join them to form a cuboid as shown in Fig. 21.5 given below:

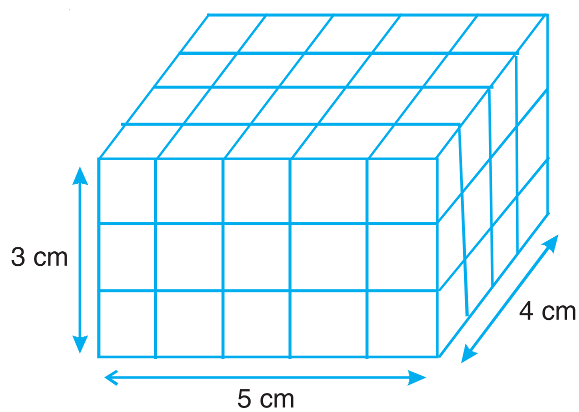


Fig. 21.5

By actually counting the unit cubes, you can see that this cuboid is made up of 60 unit cubes.

So, its volume = 60 cubic cm or 60cm^3 (Because volume of 1 unit cube, in this case, is 1 cm^3)

Also, you can observe that length × breadth × height $= 5\times4\times3$ cm^3

$= 60$ cm^3

You can form some more cuboids by joining different number of unit cubes and find their volumes by counting the unit cubes and then by the product of length, breadth and height. Everytime, you wll find that

Volume of a cuboid = length × breadth × height

or volume of a cuboid = lbh

Further, as cube is a special case of cuboid in which $l = b = h$, we have;

volume of a cube of side $a = a\times a\times a\times = a^3$.

We now take some examples to explain the use of these formulae.

Example 21.1: Length, breadth and height of cuboid are 4 cm, 3 cm and 12 cm respectively. Find

(i) surface area (ii) volume and (iii) diagonal of the cuboid.

Solution: (i) Surface area of the cuboid

$= 2\,(lb + bh + hl)$

$= 2\,(4\times3+3\times12+12\times4)\text{cm}^2$

$= 2\,(12 + 36 + 48)\text{cm}^2 = 192\text{ cm}^2$

(ii) Volume of cuboid $= lbh$

$= 4 \times 3 \times 12\text{ cm}^3 = 144\text{ cm}^2$

(iii) Diagonal of the cuboid $= \sqrt{l^2+b^2+h^2}.$

$= \sqrt{4^2+3^2+12^2}$ cm.

$= \sqrt{16+9+144}$ cm.

$= \sqrt{169}$ cm $= 13$ cm

Example 21.2: Find the volume of a cuboidal stone slab of length 3m, breadth 2m and thickness 25cm.

Solution : Here, $l = 3$m, $b = 2$m and

$$h = 25\text{cm} = \frac{25}{100} = \frac{1}{4}m$$

(Note that here we have thickness as the third dimension in place of height)

So, required volume $= lbh$

$$= 3\times2\times\frac{1}{4}\text{ m}^3 = 1.5\text{m}^3$$

Example 21.3 : Volume of a cube is 2197 cm³. Find its surface area and the diagonal.

Solution: Let the edge of the cube be a cm.

So, its volume $= a^3\text{ cm}^3$

Therefore, from the question, we have :

$a^3 = 2197$

or $a^3 = 13\times13\times13$

So, $a = 13$

i.e., edge of the cube = 13 cm

Now, surface area of the cube = $6a^2$

$$= 6 \times 13 \times 13 \text{ cm}^2$$

$$= 1014 \text{ cm}^2$$

Its diagonal = $a\sqrt{3}$ cm = $13\sqrt{3}$ cm

Thus, surface area of the cube is 1014 cm^2 and its diagonal is $13\sqrt{3}$ cm.

Example 21.4 : The length and breadth of a cuboidal tank are 5m and 4m respectively. If it is full of water and contains 60 m^3 of water, find the depth of the water in the tank.

Solution: let the depth be d metres

So, volume of water in the tank

$$= l \times b \times \text{h}$$

$$= 5 \times 4 \times \text{d m}^3$$

Thus, according to the question,

$$5 \times 4 \times d = 60$$

or $$d = \frac{60}{5 \times 4} \text{ m} = 3\text{m}$$

So, depth of the water in the tank is 3m.

Note: Volume of a container is usually called its **capacity**. Thus, here it can be said that capacity of the tank is 60m^3. Capacity is also expressed in terms of litres, where 1 litre = $\frac{1}{1000}\text{ m}^3$, i.e., 1m^3 = 1000 litres.

So, it can be said that capacity of the tank is 60 × 1000 litre = 60 kilolitres.

Example 21.5 : A wooden box 1.5m long, 1.25 m broad, 65 cm deep and open at the top is to be made. Assuming the thickness of the wood negligible, find the cost of the wood required for making the box at the rate of ₹ 200 per m^2.

Solution : Surface area of the wood required

$$= lb + 2bh + 2hl \text{ (Because the box is open at the top)}$$

$$= (1.5 \times 1.25 + 2 \times 1.25 \times \frac{65}{100} + 2 \times \frac{65}{100} \times 1.5)\text{m}^2$$

$= (1.875 + \frac{162.5}{100} + \frac{195}{100})\ m^2$

$= (1.875 + 1.625 + 1.95) m^2 = 5.450\ m^2$

So, cost of the wood at the rate of ₹ 200 per m^2

$= ₹\ 200 \times 5.450$

$= ₹\ 1090$

Example 21.6 : A river 10m deep and 100m wide is flowing at the rate of 4.5 km per hour. Find the volume of the water running into the sea per second from this river.

Solution : Rate of flow of water = 4.5 km/h

$= \frac{4.5 \times 1000}{60 \times 60}$ metres per second

$= \frac{4500}{3600}$ metres per second

$= \frac{5}{4}$ metres per second

Threfore, volume of the water running into the sea per second = volume of the cuboid

$= l \times b \times h$

$= \frac{5}{4} \times 100 \times 10\ m^3$

$= 1250\ m^3$

Example 21.7: A tank 30m long, 20m wide and 12 m deep is dug in a rectangular field of length 588 m and breadth 50m. The earth so dug out is spread evenly on the remaining part of the field. Find the height of the field raised by it.

Solution: Volume of the earth dug out = volume of a cuboid of dimensions 30 m × 20 m × 12 m

$= 30 \times 20 \times 12\ m^3 = 7200\ m^3$

Area of the remaining part of the field

= Area of the field - Area of the top surface of the tank

$= 588 \times 50\ m^2 - 30 \times 20\ m^2$

$= 29400\ m^2 - 600\ m^2$

$= 28800\ m^2$

Therefore, height of the field raised

$$= \frac{\text{Volume of earth dug out}}{\text{Area of the remaining part of the field}}$$

$$= \frac{7200}{28800}\text{ m} = \frac{1}{4}\text{ m} = 25\text{ cm}$$

Example 21.8: Length, breadth and height of a room are 7m, 4m and 3m respectively. It has a door and a window of dimensions $2\text{ m} \times 1\frac{1}{2}\text{ m}$ and $1\frac{1}{2}\text{ m} \times 1\text{ m}$ respectively. Find the cost of white washing the walls and ceiling of the room at the rate of ₹ 4 per m^2.

Solution: Shape of the room is that of a cuboid.

Area to be white washed = Area of four walls

+ Area of the ceiling

– Area of the door – Area of the window.

Area of the four walls = $l\times h+b\times h+l\times h+b\times h$

$= 2(l+b)\times h$

$= 2(7+4)\times 3\text{ m}^2 = 66\text{m}^2$

Area of the ceiling = $l\times b$

$= 7\times 4\text{ m}^2 = 28\text{m}^2$

So, area to be white washed = $66\text{ m}^2 + 28\text{ m}^2 - 2\times 1\frac{1}{2}\text{ m}^2 - 1\frac{1}{2}\times 1\text{m}^2$

$$= 94\text{ m}^2 - 3\text{ m}^2 - \frac{3}{2}\text{ m}^2$$

$$= \frac{(188-6-3)}{2}\text{ m}^2$$

$$= \frac{179}{2}\text{ m}^2$$

Therefore, cost of white-washing at the rate of ₹ 4 per m^2

$$= ₹\ 4 \times \frac{179}{2} = ₹\ 358$$

Note: You can directly use the relation area of four walls = $2(l+b)\times h$ as a formula]

CHECK YOUR PROGRESS 21.1

1. Find the surface area and volume of a cuboid of length 6m, breadth 3m and height 2.5m.
2. Find the surface area and volume of a cube of edge 3.6 cm
3. Find the edge of a cube whose volume is 3375 cm^3. Also, find its surface area.
4. The external dimensions of a closed wooden box are 42 cm $\times$ 32 cm $\times$ 27 cm. Find the internal volume of the box, if the thickness of the wood is 1cm.
5. The length, breadth and height of a godown are 12m, 8m and 6 metres respectively. How many boxes it can hold if each box occupies 1.5 m^3 space?
6. Find the length and surface area of a wooden plank of width 3m, thickness 75 cm and volume 33.75m^3.
7. Three cubes of edge 8 cm each are joined end to end to form a cuboid. Find the surface area and volume of the cuboid so formed.
8. A room is 6m long, 5m wide and 4m high. The doors and windows in the room occupy 4 square metres of space. Find the cost of papering the remaining portion of the walls with paper 75cm wide at the rate of ₹ 2.40 per metre.
9. Find the length of the longest rod that can be put in a room of dimensions 6m $\times$ 4m $\times$3m.

21.3 RIGHT CIRCULAR CYLINDER

Let us rotate a rectangle ABCD about one of its edges say AB. The solid generated as a result of this rotation is called a **right circular cylinder** (See Fig. 21.6). In daily life, we come across many solids of this shape such as water pipes, tin cans, drumes, powder boxes, etc.

It can be seen that the two ends (or bases) of a right circular cylinder are congruent circles. In Fig. 21.6, A and B are the centres of these two circles of radii AD (= BC). Further, AB is perpendicular to each of these circles.

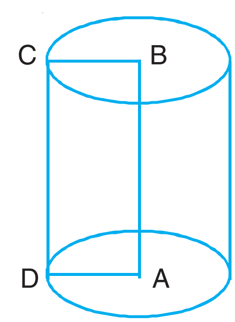


Fig. 21.6

Here, AD (or BC) is called the **base radius** and AB is called the **height** of the cylinder.

It can also be seen that the surface formed by two circular ends are **flat** and the remaining surface is **curved**.

Surface Area

Let us take a hollow cylinder of radius r and height h and cut it along any line on its curved surface parallel to the line segment joining the centres of the two circular ends (see Fig. 21.7(i)].We obtain a rectangle of length $2\pi r$ and breadth h as shown in Fig. 21.7 (ii). Clearly, area of this rectangle is equal to the area of the curved surface of the cylinder.

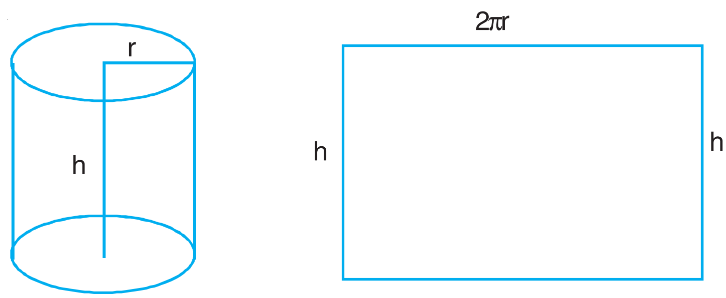


Fig. 21.7

So, curved surface area of the cylinder

$= \text{area of the rectangle}$

$= 2\pi r \times h = 2\pi rh.$

In case, the cylinder is closed at both the ends, then the total surface area of the cylinder

$= 2\pi rh + 2\pi r^2$

$= 2\pi r(r + h)$

Volume

In the case of a cuboid, we have seen that its volume $= l \times b \times h$

$= \text{area of the base} \times \text{height}$

Extending this rule for a right circular cylinder (assuming it to be the sum of the infininte number of small cuboids), we get : **Volume of a right circular cylinder**

$= \text{Area of the base} \times \text{height}$

$= \pi r^2 \times h$

$= \pi r^2 h$

We now take some examples to illustrate the use of these formulae; (In all the problems in this lesson, we shall take the value of $\pi = 22/7$, unless stated otherwise)

Example 21.9: The radius and height of a right circular cylinder are 7cm and 10cm respectively. Find its

(i) curved surface area

(ii) total surface area, and the

(iii) volume

Solution : (i) curved surface area = $2\pi rh$

$$= 2\times\frac{22}{7}\times 7\times 10\ cm^2 = 440\ cm^2$$

(ii) total surface area $= 2\pi rh + 2\pi r^2$

$$= (2\times\frac{22}{7}\times 7\times 10 + 2\times\frac{22}{7}\times 7\times 7)\ cm^2$$

$$= 440\ cm^2 + 308\ cm^2 = 748\ cm^2$$

(iii) volume $= \pi r^2 h$

$$= \frac{22}{7}\times 7\times 7\times 10 cm^3$$

$$= 1540\ cm^3$$

Example 21.10: A hollow cylindrical metallic pipe is open at both the ends and its external diameter is 12 cm. If the length of the pipe is 70 cm and the thickness of the metal used is 1 cm, find the volume of the metal used for making the pipe.

Solution: Here, external radius of the pipe

$$= \frac{12}{2}\ cm = 6cm$$

Therefore, internal radius = (6–1) = 5 cm (As thickness of metal = 1 cm)

Note that here virtually two cylinders have been formed and the volume of the metal used in making the pipe.

= Volume of the external cylinder – Volume of the internal cylinder

$= \pi r_1^2 h - \pi r_2^2 h$ (where r_1 and r_2 are the external and internal radii and h is the length of each cylinder .

$$= \left(\frac{22}{7}\times 6\times 6\times 70 - \frac{22}{7}\times 5\times 5\times 70\right) cm^3$$

$$= 22\times 10\times (36 - 25) cm^3$$

$$= 2420\ cm^3$$

Example 21.11: Radius of a road roller is 35 cm and it is 1 metre long. If it takes 200 revolutions to level a playground, find the cost of levelling the ground at the rate of ₹ 3 per m^2.

Solution: Area of the playground levelled by the road roller in one revolution

= curved surface area of the roller

$= 2\pi rh = 2 \times \frac{22}{7} \times 35 \times 100 \text{ cm}^2$ (r = 35 cm, h = 1 m = 100 cm)

$= 22000 \text{ cm}^2$

$= \frac{22000}{100 \times 100} \text{ m}^2$

(since 100 cm = 1 m, so 100 cm × 100 cm = 1 m × 1 m)

$= 2.2 \text{ m}^2$

Therefore, area of the playground levelled in 200 revolutions = $2.2 \times 200 \text{ m}^2 = 440 \text{ m}^2$

Hence, cost of levelling at the rate of ₹ 3 per m^2 = ₹ 3 × 440 = ₹ 1320.

Example 21.12: A metallic solid of volume 1 m^3 is melted and drawn into the form of a wire of diameter 3.5 mm. Find the length of the wire so drawn.

Solution: Let the length of the wire be x mm

You can observe that wire is of the shape of a right circular cylinder.

Its diameter = 3.5 mm

So, its radius = $\frac{3.5}{2} \text{ mm} = \frac{35}{20} = \frac{7}{4} \text{ mm}$

Also, length of wire will be treated as the height of the cylinder.

So, volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times x \text{ mm}^3$$

But the wire has been drawn from the metal of volume 1 m^3

Therefore, $\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times \frac{x}{1000000000} = 1$ (since 1 m = 1000 mm)

or $x = \frac{1 \times 7 \times 4 \times 4 \times 1000000000}{22 \times 7 \times 7}$ mm.

$$= \frac{16000000000}{154}$$

Thus, length of the wire $= \frac{16000000000}{154}$ mm

$$= \frac{16000000000}{154000} \text{ m}$$

$$= \frac{16000000}{154} \text{ m} = 103896 \text{ m (approx)}$$

CHECK YOUR PROGRESS 21.2

1. Find the curved surface area, total surface area and volume of a right circular cylinder of radius 5 m and height 1.4 m.
2. Volume of a right circular cylinder is 3080 cm^3 and radius of its base is 7 cm. Find the curved surface area of the cylinder.
3. A cylindrical water tank is of base diameter 7 m and height 2.1 m. Find the capacity of the tank in litres.
4. Length and breadth of a paper is 33 cm and 16 cm respectively. It is folded about its breadth to form a cylinder. Find the volume of the cylinder.
5. A cylindrical bucket of base diameter 28 cm and height 12 cm is full of water. This water is poured in to a rectangular tub of length 66 cm and breadth 28 cm. Find the height to which water will rise in the tub.
6. A hollow metallic cylinder is open at both the ends and is of length 8 cm. If the thickness of the metal is 2 cm and external diameter of the cylinder is 10 cm, find the whole curved surface area of the cylinder (use $\pi = 3.14$).

   [**Hint:** whole curved surface = Internal curved surface + External curved surface]

21.4 RIGHT CIRCULAR CONE

Let us rotate a right triangle ABC right angled at B about one of its side AB containing the right angle. The solid generated as a result of this rotation is called a **right circular cone** (see Fig. 21.8). In daily life, we come across many objects of this shape, such as Joker's cap, tent, ice cream cones, etc.

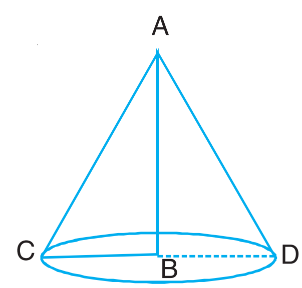


Fig. 21.8

It can be seen that end (or base) of a right circular cone is a circle. In Fig. 21.8, BC is the **radius** of the base with centre B and AB is the **height** of the cone and it is perpendicular to the base. Further, A is called the **vertex** of the cone and AC is called its **slant height**. from the Pythagoras Theorem, we have

$$\text{slant height} = \sqrt{\text{radius}^2 + \text{height}^2}$$

or $l = \sqrt{r^2 + h^2}$, where r, h and l are respectively the base radius, height and slant height of the cone.

You can also observe that surface formed by the base of the cone is **flat** and the remaining surface of the cone is **curved**.

Surface Area

Let us take a hollow right circular cone of radius r and height h and cut it along its slant height. Now spread it on a piece of paper. You obtain a sector of a circle of radius l and its arc length is equal to $2\pi r$ (Fig. 21.9).

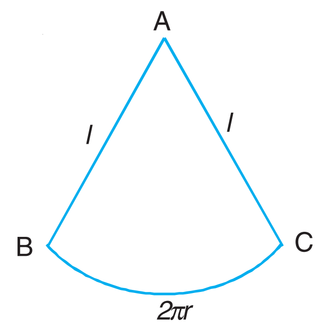


Fig. 21.9

Area of this sector =

$$\frac{\text{Arc length of the sector}}{\text{Circumference of the circle with radius } l} \times \text{Area of circle with radius } l$$

$$= \frac{2\pi r}{2\pi l} \times \pi l^2 = \pi r l$$

Clearly, curved surface of the cone = Area of the sector

$$= \boldsymbol{\pi r l}$$

If the area of the base is added to the above, then it becomes the total surface area.

So, total surface area of the cone = $\pi r l + \pi r^2$

$$= \boldsymbol{\pi r(l + r)}$$

Volume

Take a right circular cylinder and a right circular cone of the same base radius and same height. Now, fill the cone with sand (or water) and pour it in to the cylinder. Repeat the process three times. You will observe that the cylinder is completely filled with the sand (or water). It shows that volume of a cone with radius r and height h is one third the volume of the cylinder with radius r and height h.

So, **volume of a cone** = $\frac{1}{3}$ **volume of the cylinder**

$$= \frac{1}{3}\pi r^2 h$$

Now, let us consider some examples to illustrate the use of these formulae.

Example 21.13: The base radius and height of a right circular cone is 7 cm and 24 cm. Find its curved surface area, total surface area and volume.

Solution: Here, r = 7 cm and h = 24 cm.

So, slant height $l = \sqrt{r^2 + h^2}$

$$= \sqrt{7 \times 7 + 24 \times 24} \text{ cm}$$

$$= \sqrt{49 + 576} \text{ cm} = 25 \text{ cm}$$

Thus, curved surface area = $\pi r l$

$$= \frac{22}{7} \times 7 \times 25 \text{ cm}^2 = 550 \text{ cm}^2$$

Total surface area = $\pi r l + \pi r^2$

$$= (550 + \frac{22}{7} \times 49) \text{ cm}^2$$

$$= (550 + 154) \text{ cm}^2 = 704 \text{ cm}^2$$

Volume = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 49 \times 24 \text{ cm}^3$

$= 1232 \text{ cm}^3$

Example 21.14: A conical tent is 6 m high and its base radius is 8 m. Find the cost of the canvas required to make the tent at the rate of ₹ 120 per m^2 (Use $\pi = 3.14$)

Solution: Let the slant height of the tent be x metres.

So, from $l = \sqrt{r^2 + h^2}$ we have,

$$l = \sqrt{36 + 64} = \sqrt{100}$$

or $l = 10$

Thus, slant height of the tent is 10 m.

So, its curved surface area = $\pi r l$

$$= 3.14 \times 8 \times 10 \text{ cm}^2 = 251.2 \text{ cm}^2$$

Thus, canvas required for making the tent = 251.2 m^2

Therefore, cost of the canvas at ₹ 120 per m^2

$$= ₹\ 120 \times 251.2$$

$$= ₹\ 30144$$

CHECK YOUR PROGRESS 21.3

1. Find the curved surface area, total surface area and volume of a right circular cone whose base radius and height are respectively 5 cm and 12 cm.
2. Find the volume of a right circular cone of base area 616 cm^2 and height 9 cm.
3. Volume of a right circular cone of height 10.5 cm is 176 cm^3. Find the radius of the cone.
4. Find the length of the 3 m wide canvas required to make a conical tent of base radius 9 m and height 12 m (use $\pi = 3.14$).
5. Find the curved surface area of a right circular cone of volume 12936 cm^3 and base diameter 42 cm.

21.5 SPHERE

Let us rotate a semicircle about its diameter. The solid so generated with this rotation is called a **sphere**. It can also be defined as follows:

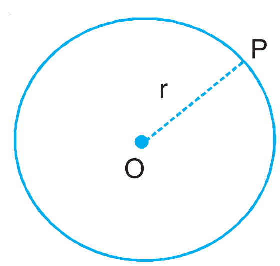


Fig. 21.10

The locus of a point which moves in space in such a way that its distance from a fixed point remains the same is called a sphere. The fixed point is called the **centre** of the sphere and the same distance is called the **radius** of the sphere (Fig. 21.10). A football, cricket ball, a marble etc. are examples of spheres that we come across in daily life.

Hemisphere

If a sphere is cut into two equal parts by a plane passing through its centre, then each part is called a hemisphere (Fig. 21.11).

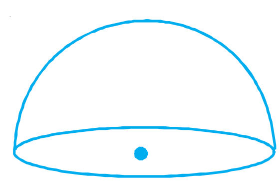
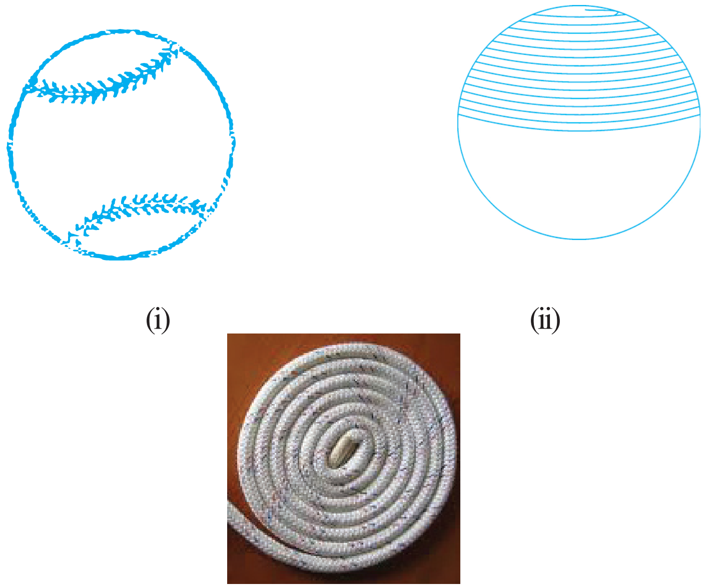


Fig. 21.11

Surface Areas of sphere and hemisphere

Let us take a spherical rubber (or wooden) ball and cut it into equal parts (hemisphere) [See Fig. 21.12(i), Let the radius of the ball be *r*. Now, put a pin (or a nail) at the top of the ball. starting from this point, wrap a string in a spiral form till the upper hemisphere is completely covered with string as shown in Fig. 21.12(ii). Measure the length of the string used in covering the hemisphere.



(i) (ii) (iii)

Fig. 21.12

Now draw a circle of radius r (i.e. the same radius as that of the ball and cover it with a similar string starting from the centre of the circle [See Fig. 21.12 (iii)]. Measure the length of the string used to cover the circle. What do you observe? You will observe that **length of the string used to cover the hemisphere is twice the length of the string used to cover the circle.**

Since the width of the two strings is the same, therefore

surface area of the hemisphere = 2 × area of the circle

$= 2\ \pi r^2$ (Area of the circle is πr^2)

So, surface area of the sphere $= 2 \times 2\pi r^2 = 4\pi r^2$

Thus, we have:

Surface area of a sphere $= 4\pi r^2$

Curved surface area of a solid hemisphere $= 2\pi r^2 + \pi r^2 = 3\pi r^2$

Where r is the radius of the sphere (hemisphere)

Volumes of Sphere and Hemisphere

Take a hollow hemisphere and a hollow right circular cone of the same base radius and same height (say r). Now fill the cone with sand (or water) and pour it into the hemisphere. Repeat the process two times. You will observe that hemisphere is completely filled with the sand (or water). It shows that volume of a hemisphere of radius r is twice the volume of the cone with same base radius and same height.

So, volume of the hemisphere $= 2 \times \frac{1}{3}\pi r^2 h$

$$= \frac{2}{3} \times \pi r^2 \times r \qquad \text{(Because } h = r\text{)}$$

$$= \frac{2}{3} \times \pi r^3$$

Therefore, volume of the sphere of radius r

$$= 2 \times \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$$

Thus, we have:

Volume of a sphere $= \frac{4}{3}\pi r^3$

and volume of a hemisphere $= \frac{2}{3}\pi r^3$,

where r is the radius of the sphere (or hemisphere)

Let us illustrate the use of these formulae through some examples:

Example 21.15: Find the surface area and volume of a sphere of diameter 21 cm.

Solution: Radius of the sphere $= \frac{21}{2}$ cm

So, its surface area $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \text{ cm}^2$$

$$= 1386 \text{ cm}^2$$

Its volume $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \text{ cm}^3 = 4851 \text{ cm}^3$$

Example 21.16: The volume of a hemispherical bowl is 2425.5 cm^3. Find its radius and surface area.

Solution: Let the radius be r cm.

So, $\frac{2}{3}\pi r^3 = 2425.5$

or $\frac{2}{3} \times \frac{22}{7} r^3 = 2425.5$

or $r^3 = \frac{3 \times 2425.5 \times 7}{2 \times 22} = \frac{21 \times 21 \times 21}{8}$

So, $r = \frac{21}{2}$, i.e. radius = 10.5 cm.

Now surface area of bowl = curved surface area = $2\pi r^2 = 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$ cm^2

= 693 cm^2

Note: As the bowl (hemisphere) is open at the top, therefore area of the top, i.e., πr^2 will not be included in its surface area.

CHECK YOUR PROGRESS 21.4

1. Find the surface area and volume of a sphere of radius 14 cm.
2. Volume of a sphere is 38808 cm^3. Find its radius and hence its surface area.
3. Diameter of a hemispherical toy is 56 cm. Find its

   (i) curved surface area

   (ii) total surface area

   (iii) volume
4. A metallic solid ball of radius 28 cm is melted and converted into small solid balls of radius 7 cm each. Find the number of small balls so formed.

LET US SUM UP

- The objects or figures that do not wholly lie in a plane are called solid (or three dimensional) objects or figures.

- The measure of the boundary constituting the solid figure itself is called its surface.
- The measure of the space region enclosed by a solid figure is called its volume.
- some solid figures have only flat surfaces, some have only curved surfaces and some have both flat as well as curved surfaces.
- Surface area of a cuboid = $2(lb + bh + hl)$ and volume of cuboid = lbh, where l, b and h are respectively length, breadth and height of the cuboid.
- Diagonal of the above cuboid is $\sqrt{l^2+b^2+h^2}$
- Cube is a special cuboid whose each edge is of same length.
- Surface area of a cube of edge a is $6a^2$ and its volume is a^3.
- Diagonal of the above cube is $a\sqrt{3}$.
- Area of the four walls of a room of dimensions l, b and $h = 2(l + b)\, h$
- Curved surface area of a right circular cylinder = $2\pi rh$; its total surface area = $2\pi rh+2\pi r^2$ and its volume = $\pi r^2 h$, where r and h are respectively the base radius and height of the cylinder.
- Curved surface area of a right circular cone is πrl, its total surface area = $\pi rl + \pi r^2$ and its volume = $\frac{1}{3}\pi r^2 h$, where r, h and l are respectively the base radius, height and slant height of the cone.
- Surface area of sphere = $4\pi r^2$ and its volume = $\frac{4}{3}\pi r^3$, where r is the radius of the sphere.
- Curved surface area of a hemisphere of radius $r = 2\pi r^2$; its total surface area = $3\pi r^2$ and its volume = $\frac{2}{3}\pi r^3$

TERMINAL EXERCISE

1. Fill in the blanks:

   (i) Surface area of a cuboid of length l, breadth b and height h = __________

   (ii) Diagonal of the cuboid of length l, breadth b and height h = ____________

   (iii) Volume of the cube of side a = ___________

(iv) Surface area of cylinder open at one end = ______, where r and h are the radius and height of the cylinder.

(v) Volume of the cylinder of radius r and height h = ___________

(vi) Curved surface area of cone = _____________, where r and l are respectively the ______ and __________ of the cone.

(vii) Surface area of a sphere of radius r = ___________

(viii) Volume of a hemisphere of radius r = ____________

2. Choose the correct answer from the given four options:

(i) The edge of a cube whose volume is equal to the volume of a cuboid of dimensions 63 cm × 56 cm × 21 cm is

(A) 21 cm (B) 28 cm (C) 36 cm (D) 42 cm

(ii) If radius of a sphere is doubled, then its volume will become how many times of the original volume?

(A) 2 times (B) 3 times (C) 4 times (D) 8 times

(iii) Volume of a cylinder of the same base radius and the same height as that of a cone is

(A) the same as that of the cone (B) 2 times the volume of the cone

(C) $\frac{1}{3}$ times the volume of the cone (D) 3 times the volume of the cone.

3. If the surface area of a cube is 96 cm^2, then find its volume.

4. Find the surface area and volume of a cuboid of length 3m, breadth 2.5 m and height 1.5 m.

5. Find the surface area and volume of a cube of edge 1.6 cm.

6. Find the length of the diagonal of a cuboid of dimensions 6 cm × 8 cm × 10 cm.

7. Find the length of the diagonal of a cube of edge 8 cm.

8. Areas of the three adjecent faces of cuboid are A, B and C square units respectively and its volume is V cubic units. Prove that $V^2 = ABC$.

9. Find the total surface area of a hollow cylindrical pipe open at the ends if its height is 10 cm, external diameter 10 cm and thickness 12 cm (use $\pi = 3.14$).

10. Find the slant height of a cone whose volume is 12936 cm^3 and radius of the base is 21 cm. Also, find its total surface area.

11. A well of radius 5.6 m and depth 20 m is dug in a rectangular field of dimensions 150 m × 70 m and the earth dug out from it is evenly spread on the remaining part of the field. Find the height by which the field is raised.

12. Find the radius and surface area of a sphere whose volume is 606.375 m^3.

13. In a room of length 12 m, breadth 4 m and height 3 m, there are two windows of dimensions 2m × 1 m and a door of dimensions 2.5 m × 2 m. Find the cost of papering the walls at the rate of ₹ 30 per m^2.

14. A cubic centimetre gold is drawn into a wire of diameter 0.2 mm. Find the length of the wire. (use $\pi = 3.14$).

15. If the radius of a sphere is tripled, find the ratio of the

    (i) Volume of the original sphere to that of the new sphere.

    (ii) surface area of the original sphere to that of the new sphere.

16. A cone, a cylinder and a hemisphere are of the same base and same height. Find the ratio of their volumes.

17. Slant height and radius of the base of a right circular cone are 25 cm and 7 cm respectively. Find its

    (i) curved surface area

    (ii) total surface area, and

    (iii) volume

18. Four cubes each of side 5 cm are joined end to end in a row. Find the surface and the volume of the resulting cuboid.

19. The radii of two cylinders are in the ratio 3 : 2 and their heights are in the ratio 7 : 4. Find the ratio of their

    (i) volumes.

    (ii) curved surface areas.

20. State which of the following statements are true and which are false:

    (i) Surface area of a cube of side a is $6a^2$.

    (ii) Total surface area of a cone is πrl, where r and l are resepctively the base radius and slant height of the cone.

    (iii) If the base radius and height of cone and hemisphere are the same, then volume of the hemisphere is thrice the volume of the cone.

    (iv) Length of the longest rod that can be put in a room of length l, breadth b and height h is $\sqrt{l^2 + b^2 + h^2}$

    (v) Surface area of a hemisphere of radius r is $2\pi r^2$.

ANSWERS TO CHECK YOUR PROGRESS

21.1

1. $81\ m^2$; $45\ m^3$
2. $77.76\ cm^2$; $46.656\ cm^3$
3. 15 cm, $1350\ cm^2$
4. $30000\ cm^3$
5. 384
6. 15 m, $117\ m^2$
7. $896\ cm^2$, $1536\ cm^3$
8. ₹ 460.80
9. $\sqrt{61}$ m

21.2

1. $44\ m^2$; $201\frac{1}{7}\ m^2$; $110\ m^3$
2. $880\ cm^2$
3. 80850 litres
4. $1386\ cm^3$
5. 4 cm
6. $401.92\ cm^2$

21.3

1. $\frac{1430}{7}\ cm^2$; $\frac{1980}{7}\ cm^2$; $\frac{2200}{7}\ cm^3$
2. $1848\ cm^3$
3. 2 cm
4. 141.3 m
5. $2310\ cm^2$

21.4

1. $2464\ cm^2$; $11498\frac{2}{3}\ cm^3$
2. 21 cm, $5544\ cm^2$
3. (i) $9928\ cm^2$ (ii) $14892\ cm^2$ (iii) $92661\frac{1}{3}\ cm^3$
4. 64

ANSWERS TO TERMINAL EXERCISE

1. (i) $2(lb + bh + hl)$ (ii) $\sqrt{l^2 + b^2 + h^2}$ (iii) a^3
   (iv) $2\pi rh + \pi r^2$ (v) $\pi r^2 h$

(vi) πrl, radius, slant height (vii) $4\pi r^2$ (vii) $\frac{2}{3}\pi r^3$

2. (i) D (ii) D (iii) D

3. 64 cm^3 4. 31.5 m^2; 11.25 m^3 5. 11.76 cm^2; 3.136 cm^3

6. $10\sqrt{2}$ cm 7. $8\sqrt{3}$ cm 8. [**Hint**: $A = l \times h$; $B = b \times h$; and $C = h \times l$]

9. 621.72 cm^2 10. 35 cm, 3696 cm^2 11. 18.95 cm

12. 21 m, 5544 m^2 13. ₹ 2610 14. 31.84 m

15. (i) 1 : 27 (ii) 1 : 9

16. 1: 3: 2

17. (i) 550 cm^2 (ii) 704 cm^2 (iii) 1232 cm^3

18. 350 cm^2; 375 cm^3

19. (i) 63 : 16 (ii) 21 : 8

20. (i) True (ii) False (iii) False
(iv) True (v) False

Secondary Course Mathematics

Practice Work-Mensuration

Maximum Marks: 25 **Time : 45 Minutes**

Instructions:

1. Answer all the questions on a separate sheet of paper.
2. Give the following informations on your answer sheet

   Name

   Enrolment number

   Subject

   Topic of practice work

   Address
3. Get your practice work checked by the subject teacher at your study centre so that you get positive feedback about your performance.

Do not send practice work to National Institute of Open Schooling

1. The measure of each side of an equilateral triangle whose area is $\sqrt{3}$ cm^2 is 1

   (A) 8 cm

   (B) 4 cm

   (C) 2 cm

   (D) 16 cm

2. The sides of a triangle are in the ratio 3 : 5 : 7. If the perimeter of the triangle is 60 cm, then the area of the triangle is 1

   (A) $60\sqrt{3}$ cm^2

   (B) $30\sqrt{3}$ cm^3

(C) $15\sqrt{3}$ cm^2

(D) $120\sqrt{3}$ cm^2

3. The area of a rhombus is 96 sq cm. If one of its diagonals is 16 cm, then length of its side is 1

(A)5 cm

(B) 6 cm

(C) 8 cm

(D) 10 cm

4. A cuboid having surface areas of three adjacent faces as a, b, c has the volume 1

(A) $\sqrt[3]{abc}$

(B) $\sqrt{abc}$

(C) abc

(D) $a^3b^3c^3$

5. The surface area of a hemispherical bowl of radius 3.5 m is 1

(A) 38.5 m^2

(B) 77 m^2

(C) 115.5 m^2

(D) 154 m^2

6. The parallel sides of a trapezium are 20 metres and 16 metres and the distance between them is 11m. Find its area. 2

7. A path 3 metres wide runs around a circular park whose radius is 9 metres. Find the area of the path. 2

8. The radii of two right circular cylinders are in the ratio 4 : 5 and their heights are in the ratio 5 : 3. Find the ratio of their volumes. 2

9. The circumference of the base of a 9 metre high wooden solid cone is 44 m. Find the volume of the cone. 2

10. Find the surface area and volume of a sphere of diameter 41 cm. 2

11. The radius and height of a right circular cone are in the ratio 5 : 12. If its volume is 314 m^3, find its slant height. (Use $\pi = 3.14$) 4

12. A field is 200 m long and 75 m broad. A tank 40 m long, 20 m broad and 10 m deep is dug in the field and the earth taken out of it, is spread evenly over the field. How much is the level of field raised? 6

MODULE 5

Trigonometry

Imagine a man standing near the base of a hill, looking at the temple on the top of the hill. Before deciding to start climbing the hill, he wants to have an approximation of the distance between him and the temple. We know that problems of this and related problems can be solved only with the help of a science called ***trigonometry.***

The first introduction to this topic was done by ***Hipparcus*** *in* ***140 B.C.,*** *when he hinted at the possibility of finding distances and heights of inaccessible objects. In* ***150 A.D. Tolemy*** *again raised the same possibility and suggested the use of a right triangle for the same. But it was* ***Aryabhatta*** *(476 A.D.) whose introduction to the name "Jaya" lead to the name "sine" of an acute angle of a right triangle. The subject was completed by* ***Bhaskaracharya*** *(1114 A.D.) while writing his work on* ***Goladhayay.*** *In that, he used the words Jaya, Kotijya and "sparshjya" which are presently used for sine, cosine and tangent (of an angle). But it goes to the credit* ***of Neelkanth Somstuvan*** *(1500 A.D.) who developed the science and used terms like elevation, depression and gave examples of some problems on heights and distance.*

In this chapter, we shall define an angle-positive or negative, in terms of rotation of a ray from its initial position to its final position, define trigonometric ratios of an acute angle of a right triangle, in terms of its sides develop some trigonometric identities, trigonometric ratios of complementary angles and solve simple problems on height and distances, using at the most two right triangles, using angles of 30°, 45° and 60°.