18

RANDOM EXPERIMENTS AND EVENTS

In day-to-day life we see that before commencement of a cricket match two captains go for a toss. Tossing of a coin is an activity and getting either a 'Head' or a "Tail' are two possible outcomes. (Assuming that the coin does not stand on the edge). If we throw a die (of course fair die) the possible outcomes of this activity could be any one of its faces having numerals, namely 1,2,3,4,5 and 6..... at the top face.

An activity that yields a result or an outcome is called an experiment. Normally there are variety of outcomes of an experiment and it is a matter of chance as to which one of these occurs when an experiment is performed. In this lesson, we propose to study various experiments and their outcomes.

OBJECTIVES

After studying this lesson, you will be able to :

- explain the meaning of a random experiments and cite examples thereof;
- explain the role of chance in such random experiments;
- define a sample space corresponding to an experiment;
- write a sample space corresponding to a given experiment; and
- differentiate between various types of events such as equally likely, mutually exclusive, exhaustive, independent and dependent events.

EXPECTED BACKGROUND KNOWLEDGE

- Basic concepts of probability

18.1 RANDOM EXPERIMENT

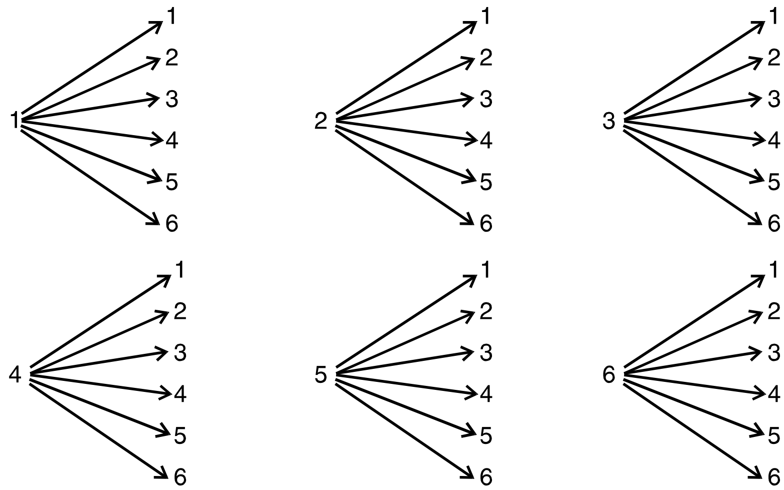
Let us consider the following activities :

(i) Toss a coin and note the outcomes. There are two possible outcomes, either a head (H) or a tail (T).

(ii) In throwing a fair die, there are six possible outcomes, that is, any one of the six faces 1,2,...... 6.... may come on top.

(iii) Toss two coins simultaneously and note down the possible outcomes. There are four possible outcomes, HH,HT,TH,TT.

(iv) Throw two dice and there are 36 possible outcomes which are represented as below :



i.e. outcomes are (1,1), (1,2), (1,3), (1,4), (1,5), (1,6)

$$(2,1),(2,2),....,(2,6)$$
$$\vdots \quad \vdots \quad \vdots$$
$$(6,1),(6,2),....,(6,6)$$

Each of the above mentioned activities fulfil the following two conditions.

(a) The activity can be repeated number of times under identical conditions.

(b) Outcome of an activity is not predictable beforehand, since the chance play a role and each outcome has the same chance of being selection. Thus, due to the chance playing a role, an activity is

(i) repeated under identical conditions, and

(ii) whose outcome is not predictable beforehand is called a random experiment.

Example 18.1 Is drawing a card from well shuffled deck of cards, a random experiment ?

Solution :

(a) The experiment can be repeated, as the deck of cards can be shuffled every time before drawing a card.

(b) Any of the 52 cards can be drawn and hence the outcome is not predictable beforehand.

Hence, this is a random experiment.

Example 18.2 Selecting a chair from 100 chairs without preference is a random experiment. Justify.

Solution :

(a) The experiment can be repeated under identical conditions.

(b) As the selection of the chair is without preference, every chair has equal chances of selection. Hence, the outcome is not predictable beforehand. Thus, it is a random experiment.

Can you think of any other activities which are not random in nature.

Let us consider some activities which are not random experiments.

(i) Birth of Manish : Obviously this activity, that is, the birth of an individual is not repeatable and hence is not a random experiment.

(ii) Multiplying 4 and 8 on a calculator.

Although this activity can be repeated under identical conditions, the outcome is always 32. Hence, the activity is not a random experiment.

18.2 SAMPLE SPACE

We throw a die once, what are possible outcomes ? Clearly, a die can fall with any of its faces at the top. The number on each of the faces is, therefore, a possible outcome. We write the set S of all possible outcomes as , S = {1, 2, 3, 4, 5, 6}

Again, if we toss a coin, the possible outcomes for this experiment are either a head or a tail. We write the set S of all possible outcomes as , S = {H, T}.

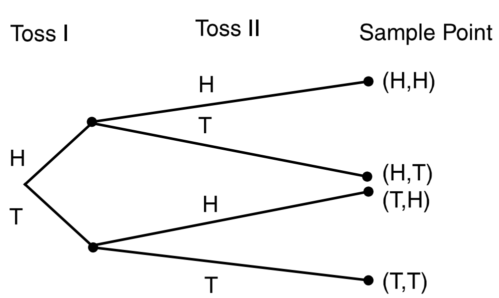
The set S associated with an experiment satisfying the following properties :

(i) each element of S denotes a possible outcome of the experiment.

(ii) any trial results in an outcome that corresponds to one and only one element of the set S is called the sample space of the experiment and the elements are called sample points. Sample space is generally denoted by S.

Example 18.3 Write the sample space in two tosses of a coin.

Solution : Let H denote a head and T denote a tail in the experiment of tossing of a coin.



S = { (H, H), (H, T), (T, H), (T, T) }.

Note : If two coins are tossed simultaneously then the sample space S can be written as

S = { H H, H T, T H, T T}.

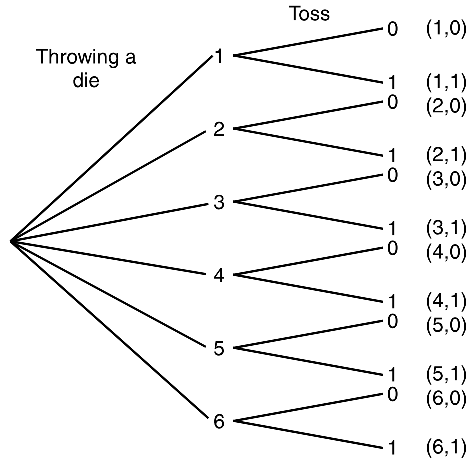
Example 18.4 Consider an experiment of rolling a fair die and then tossing a coin.

Write the sample space.

Solution : In rolling a die possible outcomes are 1, 2, 3, 4, 5 and 6. On tossing a coin the possible outcomes are either a head or a tail. Let H (head) = 0 and T (tail) = 1.

MODULE - V
Statistics and Probability

Notes

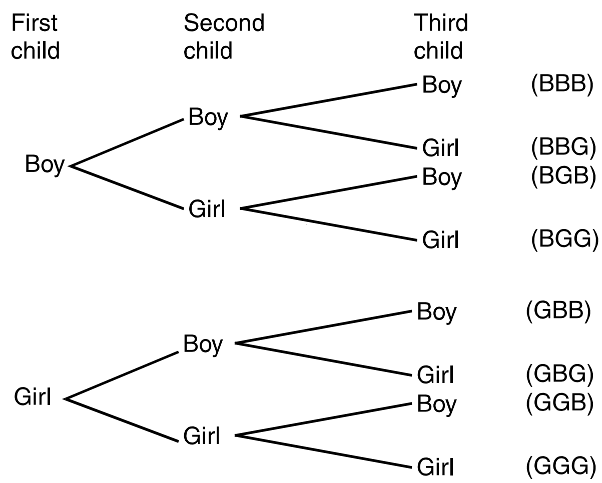


S = {(1, 0), (1, 1), (2, 0), (2, 1),(3, 0), (3, 1),(4, 0), (4, 1), (5, 0), (5, 1), (6, 0), (6, 1)}

$\therefore$ $n(S) = 6 \times 2 = 12$

Example 18.5 Suppose we take all the different families with exactly 3 children. The experiment consists in asking them the sex (or genders) of the first, second and third chid. Write down the sample space.

Solution : Let us write 'B' for boy and 'G' for girl and construct the following tree diagram.



The sample space is

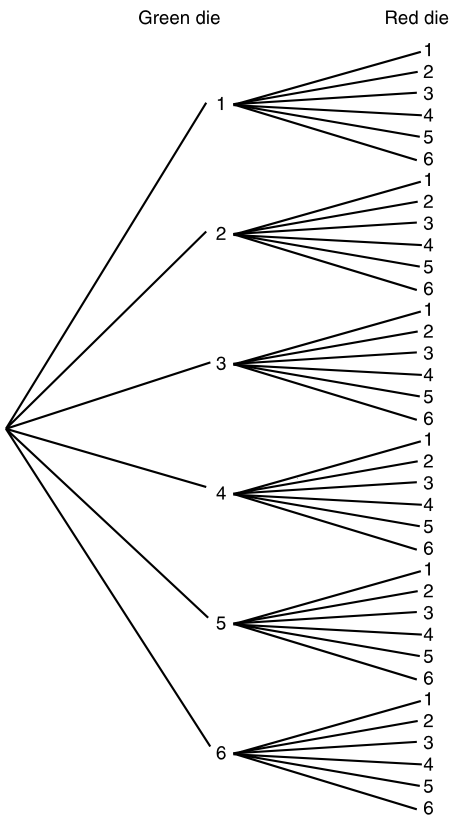
S = {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}

The advantage of writing the sample space in the above form is that a question such as "Was the second child a girl" ? or " How many families have first child a boy ?" and so forth can be answered immediately.

$$n(S) = 2 \times 2 \times 2 = 8$$

Example 18.6 Consider an experiment in which one die is green and the other is red. When these two dice are rolled, what will be the sample space ?

Solution : This experiment can be displayed in the form of a tree diagram, as shown below :



Let g_i and r_j denote, the number that comes up on the green die and red die respectively. Then an out-come can be represented by an ordered pair (g_i, r_j), where i and j can assume any of the values 1, 2, 3, 4, 5, 6.

Thus, a sample space S for this experiment is the set, $S = \{(g_i, r_j) : 1 \le i \le 6, 1 \le j \le 6\}$.

Also, notice that the multiplication principle (principle of counting) shows that the number of elements in S is 36, since there are 6 choices for g and 6 choices for r, and $6 \times 6 = 36$

$$\therefore n(S) = 36$$

Example 18.7 Write the sample space for each of the following experiments :

(i) A coin is tossed three times and the result at each toss is noted.

(ii) From five players A, B, C, D and E , two players are selected for a match.

(iii) Six seeds are sown and the number of seeds germinating is noted.

(iv) A coin is tossed twice. If the second throw results in a head, a die is thrown, otherwise a coin is tossed.

Solution :

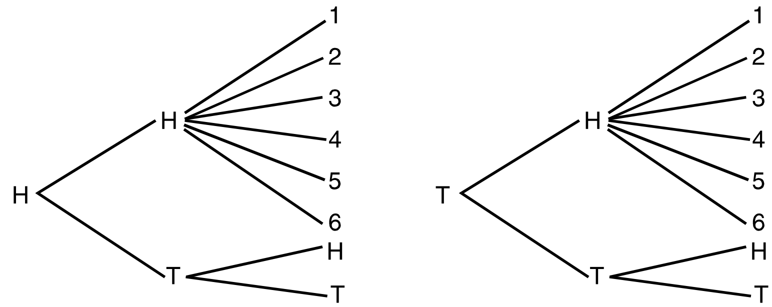
(i) S = { TTT, TTH, THT, HTT, HHT, HTH, THH, HHH}

number of elements in the sample space is $2 \times 2 \times 2 = 8$

(ii) S= { AB, AC, AD, AE, BC, BD, BE, CD, CE, DE}. Here n (S) = 10

(iii) S = { 0, 1, 2, 3, 4, 5, 6}. Here n (S) = 7

(iv) This experiment can be displayed in the form of a tree-diagram as shown below :



Thus S = {HH1, HH2, HH3, HH4, HH5, HH6, HTH, HTT, TH1, TH2,TH3,TH4, TH5, TH6, TTH, TTT}

i.e. there are 16 outcomes of this experiment.

18.3. DEFINITION OF VARIOUS TERMS

Event : Let us consider the example of tossing a coin. In this experiment, we may be interested in 'getting a head'. Then the outcome 'head' is an event.

In an experiment of throwing a die, our interest may be in, 'getting an even number'. Then the outcomes 2, 4 or 6 constitute the event. We have seen that an experiment which, though repeated under identical conditions, does not give unique results but may result in any one of the several possible outcomes, which constitute the sample space.

Some outcomes of the sample space satisfy a specified description, which we call an 'event'.

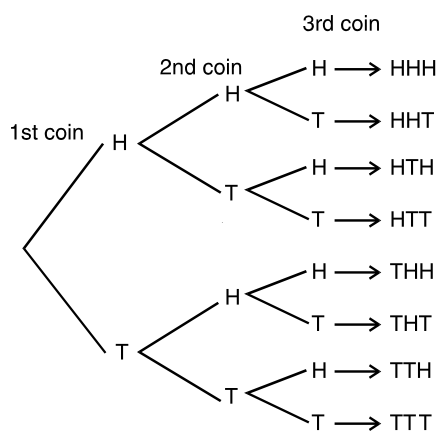
We often use the capital letters A, B, C etc. to represent the events.

Example 18.8 Let E denote the experiment of tossing three coins at a time. List all possible outcomes and the events that

(i) the number of heads exceeds the number of tails.

(ii) getting two heads.

Solution :



The sample space S is

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$$= \{ w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8 \} \text{ (say)}$$

If E_1 is the event that the number of heads exceeds the number of tails, and E_2 the event getting two heads. Then

$$E_1 = \{ w_1, w_2, w_3, w_5 \}$$

and $$E_2 = \{ w_2, w_3, w_5 \}$$

18.3.1 Equally Likely Events

Outcomes of a trial are said to be equally likely if taking into consideration all the relevant evidences there is no reason to expect one in preference to the other.

Examples :

(i) In tossing an unbiased coin, getting head or tail are equally likely events.

(ii) In throwing a fair die, all the six faces are equally likely to come.

(iii) In drawing a card from a well shuffled deck of 52 cards, all the 52 cards are equally likely to come.

18.3.2 Mutually Exclusive Events

Events are said to be mutually exclusive if the happening of any one of the them preludes the happening of all others, i.e., if no two or more of them can happen simultaneously in the same trial.

Examples :

(i) In throwing a die all the 6 faces numbered 1 to 6 are mutually exclusive. If any one of these faces comes at the top, the possibility of others, in the same trial is ruled out.

(ii) When two coins are tossed, the event that both should come up tails and the event that there must be at least one head are mutually exclusive.

Mathematically events are said to be mutually exclusive if their intersection is a null set (i.e., empty)

18.3.3 Exhaustive Events

If we have a collection of events with the property that no matter what the outcome of the experiment, one of the events in the collection must occur, then we say that the events in the collection are exhaustive events.

For example, when a die is rolled, the event of getting an even number and the event of getting an odd number are exhaustive events. Or when two coins are tossed the event that at least one head will come up and the event that at least one tail will come up are exhaustive events.

Mathematically a collection of events is said to be exhaustive if the union of these events is the complete sample space.

18.3.4 Independent and Dependent Events

A set of events is said to be independent if the happening of any one of the events does not affect the happening of others. If, on the other hand, the happening of any one of the events influence the happening of the other, the events are said to be dependent.

Examples :

(i) In tossing an unbiased coin the event of getting a head in the first toss is independent of getting a head in the second, third and subsequent throws.

(ii) If we draw a card from a pack of well shuffled cards and replace it before drawing the second card, the result of the second draw is independent of the first draw. But, however, if the first card drawn is not replaced then the second card is dependent on the first draw (in the sense that it cannot be the card drawn the first time).

CHECK YOUR PROGRESS 18.1

1. Selecting a student from a school without preference is a random experiment. Justify.
2. Adding two numbers on a calculator is not a random experiment. Justify.
3. Write the sample space of tossing three coins at a time.
4. Write the sample space of tossing a coin and a die.
5. Two dice are thrown simultaneously, and we are interested to get six on top of each of the die. Are the two events mutually exclusive or not ?
6. Two dice are thrown simultaneously. The events A, B, C, D are as below :

   A : Getting an even number on the first die.

   B : Getting an odd number on the first die.

   C : Getting the sum of the number on the dice < 7.

   D : Getting the sum of the number on the dice > 7.

   State whether the following statements are True or False.

   (i) A and B are mutually exclusive.

   (ii) A and B are mutually exclusive and exhaustive.

   (iii) A and C are mutually exclusive.

   (iv) C and D are mutually exclusive and exhaustive.
7. A ball is drawn at random from a box containing 6 red balls, 4 white balls and 5 blue balls. There will be how many sample points, in its sample space?
8. In a single rolling with two dice, write the sample space and its elements.
9. Suppose we take all the different families with exactly 2 children. The experiment consists in asking them the sex of the first and second child.

   Write down the sample space.

LET US SUM UP

- An activity that yields a result or an outcome is called an experiment.
- An activity repeated number of times under identical conditions and outcome of activity is not predictable is called Random Experiment.
- The set of possible outcomes of a random experiment is called sample space and elements of the set are called sample points.

- Some outcomes of the sample space satisfy a specified description, which is called an Event.
- Events are said to be Equally likely, when we have no preference for one rather than the other.
- If happening of an event prevents the happening of another event, then they are called Mutually Exclusive Events.
- The total number of possible outcomes in any trial is known as Exhaustive Events.
- A set of events is said to be Independent events, if the happening of any one of the events does not effect the happening of other events, otherwise they are called dependent events.

SUPPORTIVE WEB SITES

www.math.uah.edu/stat/prob/**Events**.html

http://en.wikipedia.org/wiki/**Experiment**_(probability_theory)

TERMINAL EXERCISE

1. A tea set has four cups and saucers. If the cups are placed at random on the saucers, write the sample space.
2. If four coins are tossed, write the sample space.
3. If n coins are tossed simultaneously, there will be how many sample points ?

   [Hint : try for n = 1,2, 3,4, ...]
4. In a single throw of two dice, how many sample points are there ?

MODULE - V
Statistics and Probability

Notes

ANSWERS

CHECK YOUR PROGRESS 18.1

1. Both properties are satisfied 2. Outcome is predictable

3. $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

4. $\{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T4, T6\}$ 5. No.

6. (i) True (ii) True (ii) False (iv) True 7. 15

8.
$$\{ (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

9. {MM, MF, FM, FF}

TERMINAL EXERCISE

1.
$$\{C_1S_1, C_1S_2, C_1S_3, C_1S_4, C_2S_1, C_2S_2, C_2S_3, C_2S_4,$$
$$C_3S_1, C_3S_2, C_3S_3, C_3S_4, C_4S_1, C_4S_2, C_4S_3, C_4S_4\}$$

2. $2^4 = 16$, {HHHH, HHHT, HHTH, HTHH, HHTT, HTHT, HTTH, HTTT, THHH, THHT, THTH, THTT, TTHH, TTHT, TTTH, TTTT}

3. 2^n

4. $6^2 = 36$